

Notes for HBC_VIIb*

The HBC Model is a one-dimensional model for energy transport between two interfaces of defined temperatures. It assumes two co-existing internal fluxes, nominally radiation and convection, with differing dependencies on altitude and seeks that temperature profile for which the sum of all energy fluxes is independent of altitude.

The gray gas model describes bidirectional radiative fluxes with λ , a distance characterizing radiative absorption.

$$\begin{aligned} [\lambda D + 1] J_r^+(x) &= \varphi(x) \\ [\lambda D - 1] J_r^-(x) &= -\varphi(x) \quad (D = d/dx, \sigma = \text{Stefan-Boltzmann constant}) \\ \varphi(x) &= \sigma T^4(x) \end{aligned}$$

Convection is described by a function proportional to the gradient of temperature which vanishes at the cooler interface,

$$J_c(x) = -\kappa(x) \frac{dT}{dx} = -\kappa(x) D\psi(x)$$

For a steady state, we then seek solutions for which total flux is independent of x ,

$$D[J_r^+(x) - J_r^-(x) + J_c(x)] = D J_{tot} = 0$$

Thus,

$$\begin{aligned} -J_{tot} &= [\lambda D - 1]\varphi(x) + [\lambda D + 1]\varphi(x) - [\lambda D + 1][\lambda D - 1]\kappa D\psi(x) \\ &= 2\lambda D\varphi + \kappa D\psi - \lambda D \lambda D \kappa D \end{aligned}$$

The problem to be solved is finding that function, $T(x)$, rendering J_{tot} independent of x . The leading terms linear in κ and λ and are J_r and J_c , 'radiative' and 'convective' components of the total flux. The third term, J_{rc} , is a hybrid which remains finite at the cold boundary and considered a contribution to radiative flux.

For a particular model, κ and λ are defined by polynomials, *e.g.*

Even More Enhanced Surface Convection

$$\lambda = (a0 / \text{height}) * \text{poly}([1.0000, 0.5403, 0.1805, 0.0510]) * \text{poly}([1])$$

$$\kappa = (a1 / \text{height}) * (\text{poly}([1, -1]) * \text{poly}([1, 0, 5]))$$

The λ polynomial is $(1 + 0.5403x + 0.1805x^2 + 0.0510x^3) * (1)$ and based on the U.S. Std. Atm. density profile. The κ polynomial is $(1 - x) * (1 + 5x^2)$ so that convection ceases at $x=1$.

Other model parameters are:

$T1 = 285$ # Surface Temperature (K)

$T2 = 220$ # Tropopause Temperature (K)

$\text{height} = 10.0$ # Altitude of tropopause (km) 10 ($x=1$)

$JR0, JR1 = 100, 240$ # Radiative Fluxes at surface ($x=0$) and tropopause($x=1$)

Coefficients $a0$ and $a1$ are computed by iteration to satisfy fluxes for $JR0$ and $JR1$. All spatially varying functions, T , J , λ and κ , are expressed as polynomials. Three functions are defined corresponding to the three terms in J_{tot} . The J_r term also returns the thermal profile and its gradient.

* This version requires UTF-8 Python compatibility for Greek characters.

These three functions are called by a fourth which sums them and returns J_{tot} . The temperature profile is then found as that function minimizing deviations of J_{tot} from its mean value. To define a new model, manually edit the above parameters. Several sample functions for λ and κ are commented in the file. These, or your own, should be inserted in the code at

```
for loops in range(0, 5):
    for run in range(0, 3):
         $\lambda = (\lambda_0 / height) * poly([1.0000, 0.5403, 0.1805, 0.0510]) * poly([1])$ 
         $\kappa = (\kappa_0 / height) * (poly([1, -1]) * poly([1]))$ 
```

Console Output:

New coefficients: 1.1267 22.828 (λ_0 , κ_0)

New coefficients: 1.0817 23.307

New coefficients: 1.0752 23.388

New coefficients: 1.0756 23.382

New coefficients: 1.0757 23.382

Boundary Temperatures: 285.00K, 220.00K

Tropopause: 10.00 km

Mean Flux (W/m^2): 240.168, Std Dev: 0.086099

Surface Fluxes (W/m^2):

Total 239.651

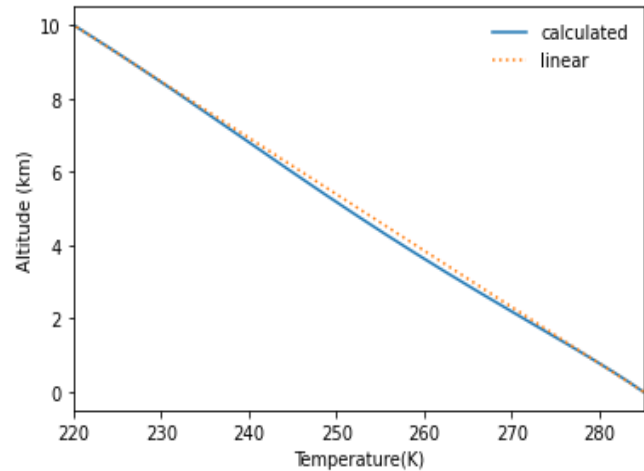
Radiation 100.006

Convection 139.645

T1 Sensitivity: 4.02 ($\text{W/m}^2/\text{K}$)

T2 Sensitivity: -3.31 ($\text{W/m}^2/\text{K}$)

6.399 seconds



Sensitivities define the total flux change resulting from independent 1K increases of each boundary. Should both boundaries change equally, e.g. unaltered lapse rates, their sum defines their joint sensitivity.

